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Entanglement and Bell violation with phase decoherence or dissipation

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Abstract. The system of an atom couples to two distinct optical cavities with decoherence is studied by making use of a dynamical algebraic method. We adopt the concurrence to characterize the entanglement between atom and cavities or between two optical cavities in the presence of the phase decoherence or dissipation. It is found that the entanglement between atom and cavities can be controlled by adjusting the detuning parameter. We show that even if the atom is initially prepared in a maximally mixed state, it can also entangle the two mode cavity fields. Finally, the Bell violation of the cavity fields is discussed, and it is shown that both the detuning and decoherence can deteriorate the maximal amount of violation of Bell inequality for two mode cavity fields during the evolution.

PACS. 03.67.Mn Entanglement production, characterization and manipulation – 03.67.-a Quantum information – 03.65.Fd Algebraic methods

1 Introduction

Quantum entanglement was first introduced by Einstein, Podolsky and Rosen (EPR) in their famous paper in 1935 [1]. Recently, it has been recognized that entanglement can be used as an important resource for quantum information processing [2]. Entanglement can exhibit the nature of a nonlocal correlation between quantum systems that have no classical interpretation. However, real quantum systems will unavoidably be influenced by surrounding environments. The interaction between the environment and quantum systems of interest can lead to decoherence. It is therefore of great importance to prevent or minimize the influence of environmental noise in the practical realization of quantum information processing. In order to prevent the effect of decoherence, several approaches have been proposed such as quantum error-correcting approach [3], quantum error-avoiding approach [4], and loop control strategies [5], etc.

The manipulation of quantum entanglement with atoms and photons in the cavity has been extensively investigated [6]. Instead of attempting to shield the system from the environmental noise, Plenio and Huelge [7] use white noise to play a constructive role and generate the controllable entanglement by incoherent sources. Similar work on this aspect has also been considered by other authors [8]. In this paper, we investigate an atom couples to two distinct optical cavities with the phase decoherence or dissipation and show how the entanglement between atom and cavities or between two optical cavities can be generated in the presence of the phase decoherence or dissipation. In Section 2, we study the system in the presence of phase decoherence by making use of the dynamical algebraical method [9,10] and find the exact solution of the master equation for the system. The exact solution is then used to discuss the influence of the phase decoherence on the probability of occupation in ground state. In Section 3, we use the concurrence to characterize the entanglement between atom and cavities or between two optical cavities by means of the exact solution for the system. It is shown that the entanglement between atom and cavities can be controlled by adjusting the detuning parameter. We proceed to calculate the generation rate of entanglement for the system and find that phase decoherence causes a larger rate than pure unitary evolution in some situations. Furthermore, we show that even if the atom is initially prepared in a maximally mixed state, it can also entangle the two mode cavity fields. In Section 4, the Bell violation of two cavity fields is investigated, and the entanglement versus Bell violation is also discussed. In Section 5, we investigate the system in the large detuning limit by extending our treatment to incorporate the dissipative processes via allowing for the radiative decay of atom as well as cavity field relaxation. A conclusion is given in Section 6.

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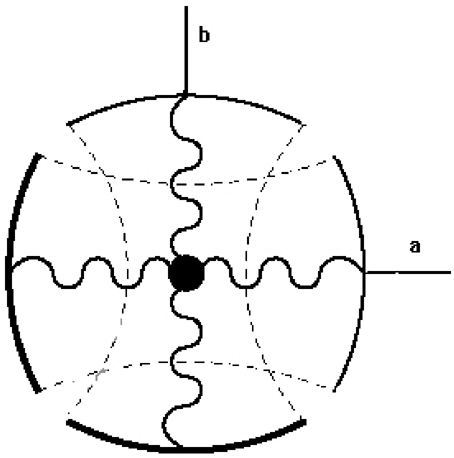

Fig. 1. Proposed experimental set-up. A two-level atom is surrounded by two optical cavities.

2 Solution of an atom couples to two distinct optical cavities with phase decoherence

We consider the situation depicted in Figure 1, that an atomic system is surrounded by two distinct optical cavities initially prepared in the vacuum state. The Hamiltonian for the system can be described by [7] ($\hbar = 1$),

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + \frac{\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + g_a(a|e\rangle\langle g| + a^\dagger|g\rangle\langle e|) + g_b(b|e\rangle\langle g| + b^\dagger|g\rangle\langle e|), \quad (1)$$

where $|e\rangle$ and $|g\rangle$ are the excited state and the ground state of the two-level atom, ω_0 is atomic transition frequency, $g_{a(b)}$ is the coupling constant of the atom to cavity modes $a(b)$, and a ($a^\dagger$), b ($b^\dagger$) are the annihilation (creation) operators of a mode of frequency ω_a and b mode of frequency ω_b, respectively. In reference [7], Plenio and Huelge use white noise as the actual driving force of the system and study numerically the entanglement between two optical cavities for the system in the resonant case. Here, we investigate analytically the entanglement between atom and cavities or between two optical cavities in the presence of phase decoherence by making use of the dynamical algebraical method. To reduce the complexity, we consider the case of $\omega_a = \omega_b = \omega$. It is easy to verify that there exists two constants of motion in Hamiltonian (1),

$$K_1 = \frac{1}{g^2}\left(g_a^2 a^\dagger a + g_b^2 b^\dagger b\right) + \frac{g_a g_b}{g^2}\left(a^\dagger b + a b^\dagger\right) + \frac{1 + |e\rangle\langle e| - |g\rangle\langle g|}{2},$$
$$K_2 = \frac{1}{g^2}\left(g_a^2 b^\dagger b + g_b^2 a^\dagger a\right) - \frac{g_a g_b}{g^2}\left(a^\dagger b + a b^\dagger\right), \quad (2)$$

where $g = \sqrt{g_a^2 + g_b^2}$. It is easily proved that the operator K_1 and K_2 commute with Hamiltonian (1). We then introduce the operators as follows

$$S_+ = \frac{(g_a a + g_b b)|e\rangle\langle g|}{g\sqrt{K_1}}, \quad S_- = \frac{(g_a a^\dagger + g_b b^\dagger)|g\rangle\langle e|}{g\sqrt{K_1}},$$
$$S_0 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|). \quad (3)$$

It can be shown that the operators S_i ($i = 0, \pm$) satisfy the following commutation relations

$$[S_0, S_\pm] = \pm S_\pm, \quad [S_+, S_-] = 2S_0, \quad (4)$$

where S_0 and $S_\pm$ are the generators of the SU(2) algebra. In terms of the SU(2) generators, we can rewrite Hamiltonian (1) as

$$H = \omega\left(K_1 + K_2 - \frac{1}{2}\right) + \Delta S_0 + g\sqrt{K_1}(S_+ + S_-), \quad (5)$$

where $\Delta = \omega_0 - \omega$ denotes detuning. Firstly, we consider the pure phase decoherence mechanism only. In this situation, the master equation governing the time evolution for the system under the Markovian approximation is given by [11]

$$\frac{d\rho}{dt} = -i[H, \rho] - \frac{\gamma}{2}[H, [H, \rho]], \quad (6)$$

where γ is the phase decoherence coefficient. Noted that the equation with the similar form has been proposed to describing the intrinsic decoherence [12]. This equation has also been utilized to describing the decoherence for a single trapped ion due to intensity and phase fluctuations in the exciting laser pulses [13]. The master equation (6) can be derived from the interaction between the system of interest and the reservoir [14]. For clarifying it, we consider the following Hamiltonian ($\hbar = 1$)

$$H_P = H + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2}m_i\omega_i^2 x_i^2\right) + H\sum_i D_i x_i + H^2 \sum_i \frac{|D_i|^2}{2m_i\omega_i^2}, \quad (7)$$

where the first term is the Hamiltonian (1) and the second term is the Hamiltonian of the reservoir. The third term describes the interaction between the system of interest and the reservoir with the coupling constant D_i, and the last one is the renormalization term. Since the Hamiltonian of system H commutes with H_P, the interaction with the reservoir can only induce phase decoherence but not dissipation. Under the Markovian approximation, the master equation (6) describing the phase decoherence of the system (1) can be obtained [14]. The formal solution of the master equation (6) can be expressed as follows [9],

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k(t)\rho(0)M^{\dagger k}(t), \quad (8)$$

where $\rho(0)$ is the density operators of the initial atom-field system and $M^k(t)$ is defined by

$$M^k(t) = H^k \exp(-iHt)\exp\left(-\frac{\gamma t}{2}H^2\right). \quad (9)$$

$$M^k(t) = \frac{1}{2}\left[f_+(K_1,K_2)\right]^k \exp\left[-if_+(K_1,K_2)t\right]\exp\left[-\frac{\gamma t}{2}\left[f_+(K_1,K_2)\right]^2\right] + \frac{1}{2}\left[f_-(K_1,K_2)\right]^k \exp\left[-if_-(K_1,K_2)t\right]\exp\left[-\frac{\gamma t}{2}\left[f_-(K_1,K_2)\right]^2\right] + \frac{1}{2}\left[\frac{\Delta}{\Omega(K_1)}(|e\rangle\langle e| - |g\rangle\langle g|) + \frac{2H_{int}}{\Omega(K_1)}\right]\left\{\left[f_+(K_1,K_2)\right]^k \exp\left[-if_+(K_1,K_2)t\right]\exp\left[-\frac{\gamma t}{2}\left[f_+(K_1,K_2)\right]^2\right] - \left[f_-(K_1,K_2)\right]^k \exp\left[-if_-(K_1,K_2)t\right]\exp\left[-\frac{\gamma t}{2}\left[f_-(K_1,K_2)\right]^2\right]\right\}, \quad (10)$$

$$\rho(t) = \frac{1}{2}\left[1+\frac{\Delta^2}{\Omega^2}+\left(1-\frac{\Delta^2}{\Omega^2}\right)\cos\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]|00\rangle\langle 00|\otimes|e\rangle\langle e| + \frac{g}{\Omega}\left\{\frac{\Delta}{\Omega}\left[1-\cos\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]+i\sin\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right\}|00\rangle\langle\varphi|\otimes|e\rangle\langle g| + \frac{g}{\Omega}\left\{\frac{\Delta}{\Omega}\left[1-\cos\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]-i\sin\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right\}|\varphi\rangle\langle 00|\otimes|g\rangle\langle e|, + \frac{2g^2}{\Omega^2}\left[1-\cos\Omega t\exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]|\varphi\rangle\langle\varphi|\otimes|g\rangle\langle g| \quad (12)$$

By means of the SU(2) dynamical algebraic structure, we obtain the explicit expression for the operator M^k

see equation (10) above

where

$$f_\pm(K_1,K_2) = \omega\left(K_1+K_2-\frac{1}{2}\right)\pm\frac{1}{2}\Omega(K_1),$$
$$\Omega(K_1) = (\Delta^2+4g^2K_1)^{1/2},$$
$$H_{int} = g_a(a|e\rangle\langle g|+a^\dagger|g\rangle\langle e|)+g_b(b|e\rangle\langle g|+b^\dagger|g\rangle\langle e|). \quad (11)$$

We assume that the cavity fields are prepared initially in vacuum state $|00\rangle$, and the atom is prepared in the excited state $|e\rangle$. The time evolution of $\rho(t)$ can be written as,

see equation (12) above

where

$$|\varphi\rangle = \frac{1}{g}(g_a|10\rangle+g_b|01\rangle), \quad \Omega = (\Delta^2+4g^2)^{1/2}. \quad (13)$$

The $|\varphi\rangle$ in equation (13) is a single-photon entangled state. Recently, much attention has been paid to investigate the preparation of the single-photon maximally entangled state [15] due to its potential applications in quantum information processing. It is noted that when the two coupling coefficients $g_a = g_b$, the state $|\varphi\rangle$ is nothing but a single-photon maximally entangled state. We then show that if a projective measurement on the atom in the $\{|e\rangle, |g\rangle\}$ basis is made, the atom will be projected on the ground state $|g\rangle$ with the probability P_g in the case of $\Delta = 0$,

$$P_g = \frac{1}{2}\left[1-\cos(2gt)\exp(-2\gamma g^2 t)\right]. \quad (14)$$

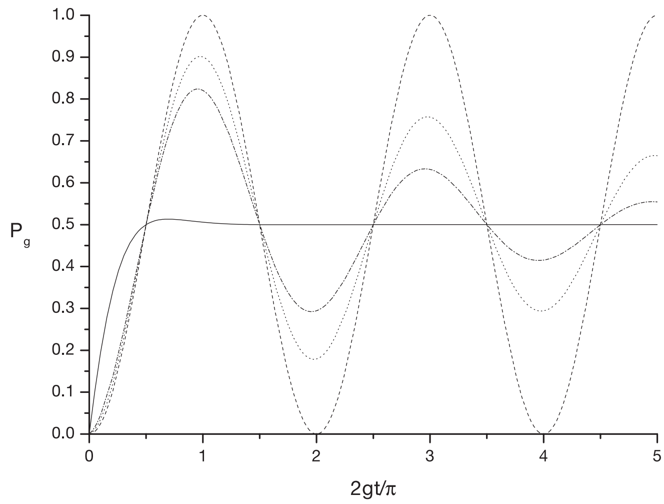

Fig. 2. The ground state probability P_g of the atom is plotted as a function of the rescaled time $2gt/\pi$ for various values of phase decoherence rate: $\gamma = 1$ (solid line), $\gamma = 0$ (dash line), $\gamma = 0.05$ (dot line) and $\gamma = 0.1$ (dash dot line) with $g_a = g_b = 1$ and $\Delta = 0$. When $t = \pi/2g$, we can see that the value of P_g with $\gamma = 0$ (dash line) is one, which implies that two distinct cavity fields are in the maximally entangled single-photon state.

If the measurement result is $|g\rangle$, the two distinct cavity fields are in the single-photon maximally entangled state $\sqrt{2}(|10\rangle + |01\rangle)/2$. In Figure 2, we plot the probability P_g as the function of time t for different values of phase decoherence rate γ. It is shown that if the decay rate γ is zero, the two distinct cavity fields are in the maximally entangled single-photon state at the time $t = \pi/2g$ with unit probability. We can also see that probability P_g in the phase decoherence case in the short time is larger than the one in the pure unitary evolution.

3 The entanglement between atom and cavities or two optical cavities

In order to quantify the degree of entanglement, several measures [16] of entanglement have been introduced for both pure and mixed quantum states. In this section, we adopt the concurrence to calculate the entanglement between atom and cavities or between two optical cavities. The concurrence related to the density operator ρ of a mixed state is defined by [17]

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{15}$$

where the λ_i ($i = 1, 2, 3, 4$) are the square roots of the eigenvalues in decreasing order of magnitude of the "spin-flipped" density operator R

$$R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{16}$$

where the asterisk indicates complex conjugation. The concurrence varies from $C = 0$ for an unentangled state to $C = 1$ for a maximally entangled state.

We first investigate the quantum entanglement between the atom and cavity modes. If we deal with the two cavity modes as system B, and the atom as system A, then $\rho(t)$ in equation (12) can be thought of as the density operator of a two-qubit mixed state. In the basis $|11\rangle_s \equiv |00\rangle \otimes |e\rangle$, $|10\rangle_s \equiv |00\rangle \otimes |g\rangle$, $|01\rangle_s \equiv |\varphi\rangle \otimes |e\rangle$, $|00\rangle_s \equiv |\varphi\rangle \otimes |g\rangle$, the explicit expression of the concurrence C_{AB} describing the entanglement between the system A and system B can be found to be,

$$C_{AB} = \frac{2g}{\Omega}\left\{\frac{\Delta^2}{\Omega^2}\left[1 - \cos\Omega t \exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]^2 + \sin^2\Omega t \exp(-\gamma t\Omega^2)\right\}^{1/2}. \tag{17}$$

From equation (17), we can see that the detuning Δ plays a key role in the quantum entanglement between the atom and cavity modes. If the decoherence rate γ is not equal to zero, the concurrence C_{AB} remains in the value $2g|\Delta|/\Omega^2$ in the limit $t \to \infty$. In the strong coupling case, i.e., $g_a, g_b \gg \Delta$, the concurrence C_{AB} of the stationary state $\rho(\infty)$ is approximately $|\Delta|/(2g)$. On the other hand, in the large detuning limit, the concurrence C_{AB} of the stationary state is approximately $2g/|\Delta|$. In Figure 3, the concurrence C_{AB} is plotted as a function of the time t and decoherence rate γ. If the decoherence rate is small enough, the entanglement between the atom and two cavities oscillates with the time. Otherwise, it rapidly evolve into a stationary value in the off-resonant case. We also show the concurrence C_{AB} as a function of the detuning parameter Δ and the decoherence rate γ at a fixed time in Figure 4. It is shown that the entanglement heavily depends on the detuning parameter, which implies one can control the entanglement via adjusting the detuning parameter. In the limit $t \to \infty$, the stationary state concurrence C_{AB} firstly increases with $|\Delta|$, and reaches the maximal value 1/2 at $|\Delta| = 2g$, then decreases with $|\Delta|$. Now,

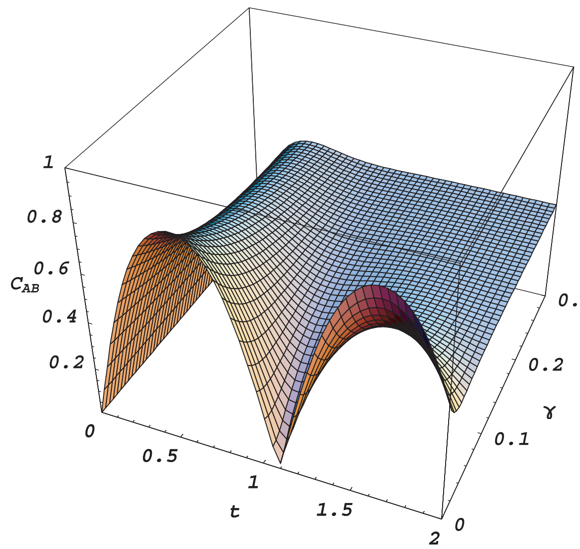

Fig. 3. The concurrence C_{AB} is plotted as a function of the time t and the phase decoherence rate γ for $g_a = g_b = 1$ and $\Delta = 5$. It is shown that, if $\gamma \ll 1$, the entanglement between the atom and two cavities oscillates with the time. Otherwise, it rapidly evolve into a stationary value.

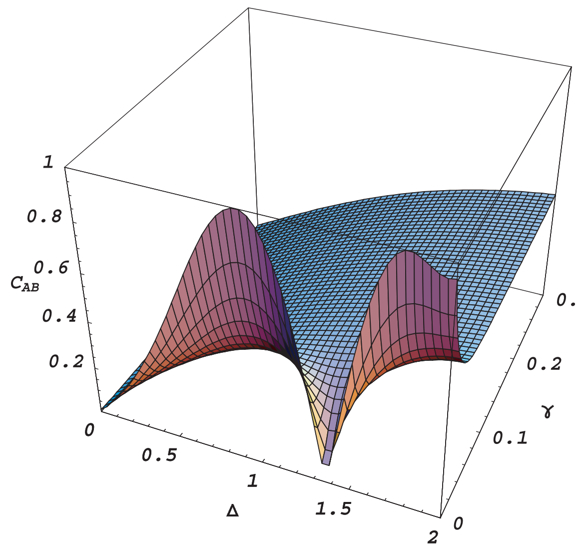

Fig. 4. The concurrence C_{AB} is plotted as a function of the detuning parameter Δ and the phase decoherence rate γ for $g_a = g_b = 1$ and $t = 10$. We can see that the entanglement heavily depends on the detuning parameter.

we turn our discussion to the resonant case, i.e. $\Delta = 0$. In this case, $C_{AB} = |\sin(2gt)|\exp(-2g^2\gamma t)$.

In reference [18], it has been proved that for any pure states of three qubits 1, 2 and 3, the entanglement is distributed following the inequality for the squared concurrence

$$C_{12}^2 + C_{13}^2 \leq C_{1(23)}^2, \tag{18}$$

where $C_{1,(23)}$ is the single-qubit concurrence defined as the concurrence between the qubit 1 and the rest of qubits (2, 3). For any mixed states of three qubits 1, 2, and 3, there is analogous inequality for the squared concurrence as follows

$$C_{12}^2 + C_{13}^2 \leq \langle C^2 \rangle_{1(23)}^{min}, \tag{19}$$

where $\langle C^2 \rangle_{1(23)}^{min}$ is the minimum of average over all possible pure state decomposition of the three qubits mixed state [18]. Here, we may expect that the pair entanglement between the atom and the $a(b)$ mode cavity field is determined by the coupling coefficient $g_a(g_b)$. It is easy to prove that there exist the simple relations,

$$C_{AB}^2 = C_a^2 + C_b^2, \tag{20}$$

and $C_a/C_b = g_a/g_b$, where $C_a(C_b)$ is the concurrence describing entanglement between the atom and the $a(b)$ mode cavity field. Thus, our results are in agreement with those obtained in reference [18].

Next, we investigate the entanglement between light fields of two distinct cavities by tracing out the atom. By tracing out the degree of freedom of the atom in density matrix $\rho(t)$ in equation (12), we obtain the reduced density matrix $\rho_B(t)$ describing the two light fields as follows,

$$\rho_B(t) = \frac{1}{2}\left[1 + \frac{\Delta^2}{\Omega^2} + \left(1 - \frac{\Delta^2}{\Omega^2}\right)\cos\Omega t \exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]|00\rangle\langle 00|, + \frac{2g^2}{\Omega^2}\left[1 - \cos\Omega t \exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]|\varphi\rangle\langle\varphi|. \tag{21}$$

Then, the concurrence C_B characterizing the entanglement of two light fields can be derived as

$$C_B = \frac{4|g_a g_b|}{\Omega^2}\left[1 - \cos\Omega t \exp\left(-\frac{\gamma t}{2}\Omega^2\right)\right]. \tag{22}$$

From equation (22), we can see that the concurrence C_B is equal to zero at time $t = 2n\pi/\Omega, (n = 0, 1, 2...)$ in the case of $\gamma = 0$. At these specific time, the two cavity modes have no pair entanglement. However, in the case with $\gamma \neq 0$, the two cavity modes is always entangled for the time $t > 0$. In Figure 5, we plot the concurrence C_B as the function of time t and decoherence rate γ. It is quite clear from Figure 5 that the entanglement between the two distinct light fields increases with the phase decoherence rate γ within the time range $2n\pi/\sqrt{\Delta^2+4g^2} \leq t < (2n+1/2)\pi/\sqrt{\Delta^2+4g^2}$ or $(2n+3/2)\pi/\sqrt{\Delta^2+4g^2} < t \leq (2n+2)\pi/\sqrt{\Delta^2+4g^2}$ $(n = 0, 1, 2, ...)$. The concurrence C_B is displayed as a function of the decoherence rate γ for three different values of the detuning parameters at a fixed time in Figure 6. From Figure 6, we see that the concurrence increases with the decoherence rate γ at a fixed time. One important point should be mentioned here. Though the entanglement increases with γ at some fixed times, it does not mean that phase decoherence can improve the maximal value of entanglement achieved during the evolution in this situation. The stationary state entanglement of the two cavity modes measured by concurrence is $4g_a g_b/(\Delta^2 + 4g^2)$. This

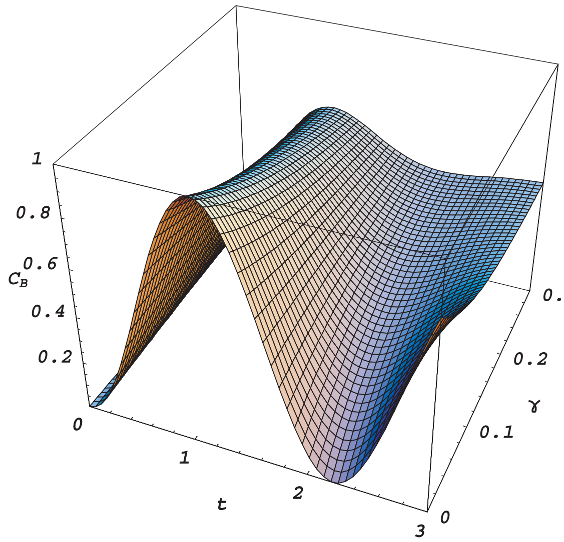

Fig. 5. The concurrence C_B is plotted as a function of the time t and the phase decoherence rate γ for $g_a = g_b = 1$ and $\Delta = 0$.

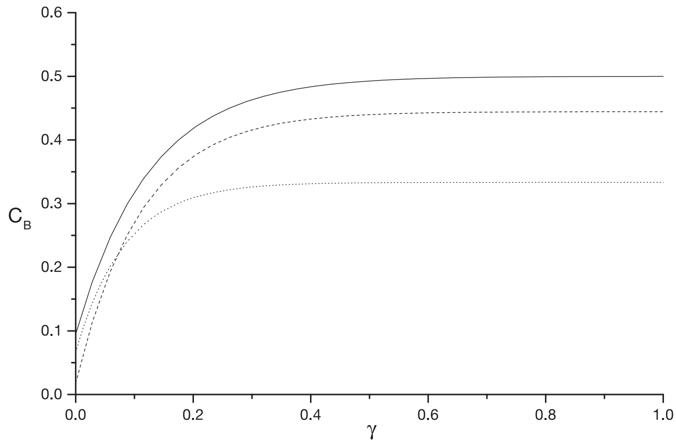

Fig. 6. The concurrence C_B is plotted as a function of the phase decoherence rate γ for various values of the detuning parameter: $\Delta = 0$ (solid line), $\Delta = 1$ (dash line) and $\Delta = 2$ (dot line) with $t = 2$ and $g_a = g_b = 1$.

means that the stationary state entanglement achieves its maximal value 1/2 in the resonant case with $g_a = g_b$.

In order to illustrate how phase decoherence can accelerate the entanglement generation of two cavity modes at some specific times, we calculate the generation rate of entanglement $\dot{C}_B$, the partial derivative of concurrence C_B over the time t. Our result indicates $\dot{C}_B$ is equal to $2|g_a g_b|\gamma$ at the initial time $t = 0$ in this case.

At the end of this section, we discuss how much entanglement between the two mode cavity fields can be achieved if the initial atom is prepared in a thermal state and the cavity fields are prepared in the vacuum states. We assume that the initial atom is in the state $\rho_A(0) = \delta|g\rangle\langle g| + (1-\delta)|e\rangle\langle e|$, where $0 \leq \delta \leq 1$, and the cavity fields are still in the vacuum state $|00\rangle$. Our calculation shows that $C'_{AB} = (1-\delta)C_{AB}$ and $C'_B = (1-\delta)C_B$. This means that even if the initial atom is prepared in a maximally mixed state $(1/2)|g\rangle\langle g| + (1/2)|e\rangle\langle e|$, it can still entangle the two mode cavity fields. In this case, the concurrence C'_B equals 1/4 in the steady state for $\Delta = 0$ and $g_a = g_b$.

4 The Bell violation of two mode cavity fields

Since the classic works of EPR [1] and Bell [19], quantum nonlocality have been recognized as a crucial notion in modern understanding of quantum world. In the present paper, we focus our attention on the Bell violation of two mode cavity fields. The reduced density matrix $\rho_B(t)$ in equation (21) describing the two cavity fields by tracing out the atom in the whole system can be regarded as a two-qubit mixed state. The most commonly discussed Bell inequality is the CHSH inequality [19,20]. The CHSH operator reads

$$\hat{B} = \vec{a} \cdot \vec{\sigma'} \otimes (\vec{b} + \vec{b'}) \cdot \vec{\sigma'} + \vec{a'} \cdot \vec{\sigma'} \otimes (\vec{b} - \vec{b'}) \cdot \vec{\sigma'}, \qquad (23)$$

where $\vec{a}, \vec{a'}, \vec{b}, \vec{b'}$ are unit vectors. The three components σ'_x, σ'_y and σ'_z of $\vec{\sigma'}$ are defined by $\sigma'_x \equiv |1\rangle\langle 0| + |0\rangle\langle 1|$, $\sigma'_y \equiv -i|1\rangle\langle 0| + i|0\rangle\langle 1|$, and $\sigma'_z \equiv |1\rangle\langle 1| - |0\rangle\langle 0|$. In the above notation, the Bell inequality reads

$$|\langle \hat{B} \rangle| \leq 2. \qquad (24)$$

The maximal amount of Bell violation of a state ρ is given by [21]

$$\mathcal{B} = 2\sqrt{\lambda + \tilde{\lambda}}, \qquad (25)$$

where λ and $\tilde{\lambda}$ are the two largest eigenvalues of $T_\rho^\dagger T_\rho$. The matrix T_ρ is determined completely by the correlation functions being a 3×3 matrix whose elements are $(T_\rho)_{nm} = \mathrm{Tr}(\rho\sigma'_n \otimes \sigma'_m)$, where, $\sigma'_1 \equiv \sigma'_x$, $\sigma'_2 \equiv \sigma'_y$, and $\sigma'_3 \equiv \sigma'_z$. We call the quantity $\mathcal{B}$ the maximal violation measure, which indicates the Bell violation when $\mathcal{B} > 2$ and the maximal violation when $\mathcal{B} = 2\sqrt{2}$. For the density operator $\rho_B(t)$ in equation (21) characterizing the time evolution of two cavity fields, $\lambda + \tilde{\lambda}$ can be written as follows

$$\lambda + \tilde{\lambda} = \varsigma + \max[\varsigma, \zeta], \qquad (26)$$

where

$$\varsigma = \frac{16g_a^2 g_b^2}{\Omega^4}(1 - e^{-\frac{\gamma t}{2}\Omega^2}\cos\Omega t)^2 \qquad (27)$$

and

$$\zeta = \left(\frac{\Delta^2}{\Omega^2} + \frac{4g^2}{\Omega^2}e^{-\frac{\gamma t}{2}\Omega^2}\cos\Omega t\right)^2. \qquad (28)$$

From equations (25–28), it is easy to see the violation of Bell inequality for two mode cavity fields. In Figure 7, the maximal amount of violation $\mathcal{B}$ is plotted as a function of time t. It is shown that both the detuning and decoherence can deteriorate the maximal amount of violation of Bell inequality for two mode cavity fields during the evolution.

Recently, Verstraete et al. investigated the relations between the violation of the CHSH inequality and the concurrence for systems of two qubits [22]. The relation

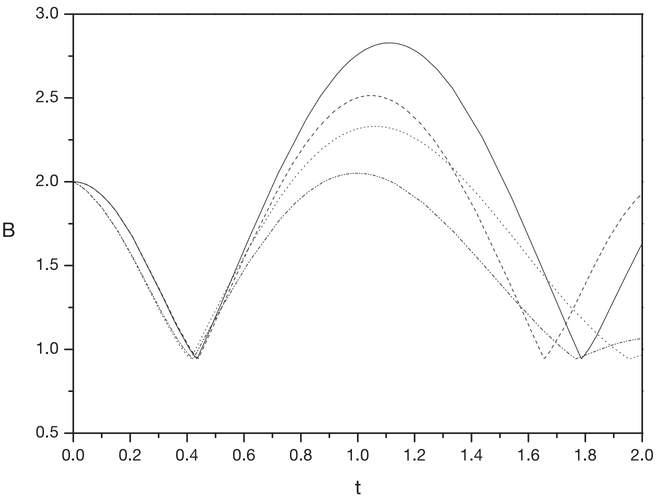

Fig. 7. The maximal amount of violation $\mathcal{B}$ is plotted as functions of time t with $g_a = g_b = 1$ for four different cases: $\Delta = 0$ and $\gamma = 0$ (solid line); $\Delta = 1$ and $\gamma = 0$ (dash line); $\Delta = 0$ and $\gamma = 0.1$ (dot line); $\Delta = 1$ and $\gamma = 0.1$ (dash dot dot line). Comparing the solid line and other lines in the regime with $\mathcal{B} > 2$ in this figure, we find that both the detuning and decoherence can deteriorate the maximal amount of violation of Bell inequality. Cautious readers maybe conjecture that the dash line may exceed the solid line in the further evolution beyond the range of this figure. This is possible. Nevertheless, our calculations show that the maximal values of the maximal violation $\mathcal{B}$ at the solid line achieved in the whole evolution is larger than the one at the dash line.

can be written as $\max[2, 2\sqrt{2}C] \leq \mathcal{B} \leq 2\sqrt{1 + C^2}$ for those states violating the CHSH inequality. They showed that the maximal value of $\mathcal{B}$ for given concurrence C is $2\sqrt{1 + C^2}$, which can be achieved by the pure states and some Bell diagonal states. If the given concurrence C is larger than $\sqrt{2}/2$, the minimal value of $\mathcal{B}$ is $2\sqrt{2}C$, which can be achieved by the maximal entangled mixed state. Furthermore, the entangled two qubits state with the concurrence $C \leq \sqrt{2}/2$ may not violate any CHSH inequality, even after all possible local filtering operations, except their Bell diagonal normal form does violate the CHSH inequalities [21]. In what follows, we show that, even though two cavity fields is always entangled during the time evolution, they violate the CHSH inequality only in the cases with some specific values of the detuning parameter Δ and the phase decoherence rate γ. In Figure 8, we plot C_M (defined as the maximal values of the concurrence C_B achieved during the decoherence process) and $\mathcal{B}_M$ (defined as the maximal values of the maximal violation $\mathcal{B}$ during the decoherence process) as the functions of the detuning parameter Δ and the phase decoherence rate γ with $g_a = g_b = 1$. It is shown that both C_M and $\mathcal{B}_M$ decrease with Δ and γ. After going beyond certain critical value of Δ and γ, the two cavity fields does not exhibit any Bell violation at any moments of the evolution. At the end of this section, we should claim that in the physical set-up considered in this paper, the violation of CHSH-inequality serves only as an evidence of entanglement. Since two cavities are not truly spatial separated,

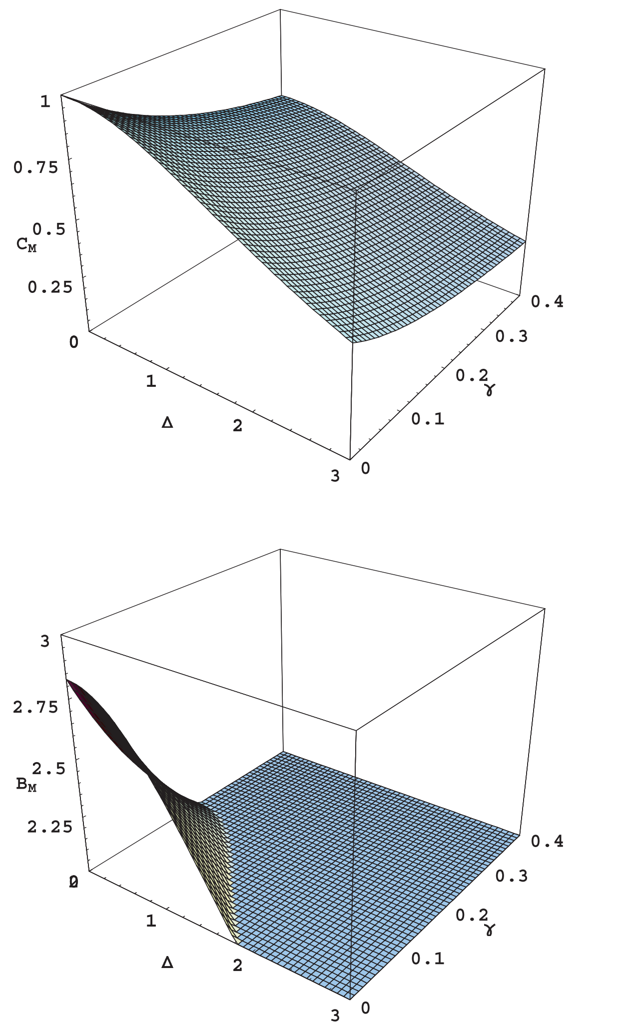

Fig. 8. C_M and $\mathcal{B}_M$ are plotted as the functions of the detuning parameter Δ and the phase decoherence rate γ with $g_a = g_b = 1$.

the Bell violation of two cavity fields discussed above can not quantify the usual nonlocality.

5 The entanglement between two optical cavities in the presence of dissipation

In this section, we investigate the system (1) in the large detuning limit by extending our previous treatment to incorporate the dissipative processes via allowing for the radiative decay of atom as well as cavity field relaxation. In reference [7], the authors had studied a similar system in the resonant case. The master equation for the total system density operator is

$$\frac{d\rho}{dt} = -i[H,\rho] + \mathcal{L}_{cav}\rho + \mathcal{L}_{at}\rho, \tag{29}$$

where

$$H = \omega a^\dagger a + \omega b^\dagger b + \frac{\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + g(a^\dagger|g\rangle\langle e| + a|e\rangle\langle g|) + g(b^\dagger|g\rangle\langle e| + b|e\rangle\langle g|). \tag{30}$$

The Liouvilleans $\mathcal{L}_{cav}\rho$ and $\mathcal{L}_{at}\rho$ are given by

$$\mathcal{L}_{cav}\rho = \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \kappa(2b\rho b^\dagger - b^\dagger b\rho - \rho b^\dagger b), \tag{31}$$

and

$$\mathcal{L}_{at}\rho = \Gamma(2|g\rangle\langle e|\rho|e\rangle\langle g| - |e\rangle\langle e|\rho - \rho|e\rangle\langle e|) \tag{32}$$

where Γ describes the spontaneous decay strength of the atom, and κ is the decay coefficient of the cavity. Here, for simplicity, we assume that two cavities have the same decay coefficient.

In the large detuning limit, there is no energy exchange between the atom and the cavities. Now, we assume that the atom is initially in the ground state. As the atom will then never be populated, we can disregard the degree of freedom of the atom in the following calculation. It is assumed that two cavities are initially in the pure product state $|1\rangle_a \otimes |0\rangle_b$. Then, the explicit analytical solution of the master equation (29) can be obtained as follows,

$$\begin{aligned}\rho(t) = {}& \rho_{11}(t)|1\rangle_{aa}\langle 1| \otimes |1\rangle_{bb}\langle 1| + \rho_{22}(t)|1\rangle_{aa}\langle 1| \otimes |0\rangle_{bb}\langle 0| \\ &+ \rho_{33}(t)|0\rangle_{aa}\langle 0| \otimes |1\rangle_{bb}\langle 1| + \rho_{44}(t)|0\rangle_{aa}\langle 0| \otimes |0\rangle_{bb}\langle 0| \\ &+ \rho_{23}(t)|1\rangle_{aa}\langle 0| \otimes |0\rangle_{bb}\langle 1| + \rho_{32}(t)|0\rangle_{aa}\langle 1| \otimes |1\rangle_{bb}\langle 0|\end{aligned} \tag{33}$$

where

$$\begin{aligned}\rho_{11}(t) &= 0, \\ \rho_{22}(t) &= \frac{1}{2}\left[1 + \cos\frac{2g^2t}{\Delta}\right]e^{-2\kappa t}, \\ \rho_{33}(t) &= +\frac{1}{2}\left[1 - \cos\frac{2g^2t}{\Delta}\right]e^{-2\kappa t} \\ \rho_{44}(t) &= 1 - e^{-2\kappa t}; \\ \rho_{23}(t) &= -\frac{i}{2}e^{-2\kappa t}\sin\frac{2g^2t}{\Delta}, \\ \rho_{32}(t) &= \frac{i}{2}e^{-2\kappa t}\sin\frac{2g^2t}{\Delta}.\end{aligned} \tag{34}$$

It is straightforward to compute analytically the concurrence for the density matrix $\rho_(t)$ in equation (33), and the concurrence $C(t)$ related to the density matrix $\rho(t)$ is found to be

$$C(t) = e^{-2\kappa t}\left|\sin\frac{2g^2t}{\Delta}\right|, \tag{35}$$

where $|x|$ gives the absolute value of x. From equation (35), we can see that the entanglement between two cavity exhibits the damped oscillation, and eventually, is completely destroyed by the cavity decay. The calculations presented in this paper can also be applied to the situation when the atom interacts with two orthogonal modes of a single cavity, which has been investigated by Rauschenbeutel et al. in the last paper of reference [15].

6 Conclusion

In this paper, we firstly investigate analytically the entanglement between atom and cavities or between two optical cavities in the presence of phase decoherence by making use of the dynamical algebraical method. It is found that the entanglement between atom and cavities can be controlled by adjusting the detuning parameter. Furthermore, we show that even if the atom is initially in a maximally mixed state, it can also entangle two mode cavity fields initially prepared in vacuum state. We also investigate the Bell violation of two mode cavity fields and find that both the detuning and decoherence can deteriorate the maximal amount of violation of Bell inequality for two mode cavity fields during the evolution. Finally, we investigate the system in the large detuning limit by extending the previous treatment to incorporate the dissipative processes via allowing for the radiative decay of atom as well as cavity field relaxation. It is shown that the entanglement between two cavity exhibits the damped oscillation, and eventually, is completely destroyed by the dissipative process of the cavities. The approach adopted here can be employed to investigate the entanglement between two optical cavities mediated by a two-level atom in those cases, in which the two mode cavity fields are initially prepared in another separable states.

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